Basic Fluid Mechanics

Text Chapters 1 & 2

Introduction
Basic Concepts and Definitions:

1. ___ - A substance which can sustain a shear force when at rest. (Molecules are closely positioned and therefore have large intermolecular forces.)

2. ___ - A substance which cannot sustain a shear force when at rest.
   
   • - A in which the molecules are free to change their position with respect to one another, but are restricted by intermolecular forces.
   
   • - A in which the molecules are free to change their positions with respect to one another, but are practically unrestricted by intermolecular forces.

3. ____________________ - The particle nature of the fluid is ignored and we assume that properties (like mass, pressure, density) can exist at a point on a per unit volume basis.

   There are $2.7 \times 10^{16}$ molecules in a cubic millimeter of air at standard conditions (STD) - $P = 101.33$ kPa (14.696 psia) and $T = 15^\circ C$ (or 59$^\circ F$).

   ($\lambda$) - The average distance a molecule travels before it collides with another molecule. For air at STD, $\lambda \sim 6 \times 10^{-6}$ cm.

   If a characteristic length of the system $\lambda$, the continuum model is acceptable.

   (\rho) - Mass per unit volume. Has units of $kg/m^3$ or $lb_{in} / ft^3$ or $lb_{sec}^2/ft^4$. (See Table A.3 pg 760 for density in the Standard Atmosphere)

   \[
   \rho = 1.2 \frac{kg}{m^3} \quad (0.00238 \frac{slug}{ft^3}) \\
   \rho = 1000 \frac{kg}{m^3} \quad (1.94 \frac{slug}{ft^3})
   \]
EXAMPLE PROBLEM 1.1:

A container with an interior volume of $10^{-3} \text{m}^3$ and a mass of 0.5 kg is put on a scale to determine the mass of the interior fluid. It is found that the combined mass of container and fluid is 1.5 kg.

What is the density of the fluid?
What fluid is it?

\[ g_c = 32.2 \frac{\text{ft} \cdot \text{lb}_m}{\text{lb}_f \cdot \text{sec}^2} \]

Used to correct between two British unit systems with different units for mass.

British Gravitational System \[ 1 \text{ slug} = 1 \text{ lb}_f \cdot \text{sec}^2/\text{ft} \]

English Engineering System \[ 1 \text{ lb}_m = \frac{1}{32.2} \text{ slug} \]

(S) - The ratio of the density of a fluid to the density of a reference fluid. (See Figure A.1, and Table A.1 and A.2 pgs 757-759 for common fluids.)
\[ S = \frac{\rho}{\rho_{ref}} \]

For liquids, the reference fluid is pure water at 4 °C (or 39.2 °F) and 101,330 N/m²; \( \rho_{H_2O} = 1000 \text{ kg/m}^3 \) (or 62.4 lbm/ft³).

(Note: engineers tend to use 60 °F as reference temperature.)

For gases, S is usually based on either hydrogen or pure air. Unfortunately, there is no agreement regarding these standards and so they must be used with care.

__________ - A fluid in which the density is assumed constant.

(See Fig. A.1 for the specific gravity of water and mercury as functions of temperature).

__________ - The perfect gas equation of state relates pressure to density and temperature via the equation \( p = \rho R T \), \( R \) - gas constant,

\[
R_{air} = 286.9 \frac{N \cdot m}{kg \cdot K} = 53.3 \frac{ft \cdot lb_f}{lb_m \cdot R}
\]

4.

__________ - Forces which are proportional to either the volume or mass of the body.

are comprised of those forces which involve action from a distance.

a) 

b) 

c) 

d) 

__________ (\( \gamma \)) - The force per unit volume which a gravitational body such as the earth exerts on
a unit volume of fluid. \( \gamma = \rho g \)

Note that \( g \) is the acceleration due to gravity (not \( g_c \)). It is \( 9.8 \text{ m/sec}^2 \) or \( 32.2 \text{ ft/sec}^2 \).

\[ \text{ Forces which are exerted at the control surface by the material outside the control volume.} \]

\[ \begin{align*}
\text{a)} \\
\text{b)} \\
\text{c)}
\end{align*} \]

\[- \text{ The force per unit area exerted perpendicular to a surface.} \]

\[ \begin{align*}
\text{S.I. units (all pressures are absolute)} \\
1 \text{ N/m}^2 = 1 \text{ pascal (Pa)}
\end{align*} \]

\[ \begin{align*}
\text{English units (gauge or absolute)} \\
1 \text{ lb/in}^2 = \text{ psi}
\end{align*} \]

is independent of direction/orientation (i.e., it's isotropic) in a fluid at rest.
Note that shear stress is a tensor, which means that it is defined only after a surface has been defined. For example, consider a channel and the fluid in contact with it. The shear stress is positive in the direction indicated.

The one-dimensional shear stress/strain relation is

$$\tau = \mu \frac{du}{dy}$$ (1.2)

Where $\tau$ is the shear stress in the x direction, $\frac{du}{dy}$ is the one-dimensional strain rate and $\mu$ is the absolute or dynamic viscosity, which has units $\text{lb}_f \text{sec}/\text{ft}^2$ or $\text{N} \cdot \text{sec}/\text{m}^2$. (See figure A.2 pg 763 for some representative values.)

Note that shear stress is a tensor, which means that it is defined only after a surface has been defined. For example, consider a channel and the fluid in contact with it. The shear stress is positive in the direction indicated.

[Diagram of shear stress in a channel.]
When considering real fluids and viscosity, it has been shown that the fluid in contact with a solid surface has the same velocity as that surface. This is called the.

A related quantity is the (ν), which has units of $ft^2/sec$ or $m^2/sec$. It is defined from the molecular viscosity by the equation

$$\nu = \frac{\mu}{\rho}$$

(See Figure A.3 pg 764 for some representative values.)

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__________ (σ) - describes the forces at the interface between a gas and a liquid (e.g., water and air) or a solid, liquid and gas (e.g., water, air and glass) and has the units of force/unit length (N/m).

Anytime there is a density discontinuity (ex. oil/H$_2$O or air/H$_2$O) surface tension exist. ($\sigma = f(P,T)$, see Table A.4 pg 761).
EXAMPLE PROBLEM 1.2:

A 60 cm wide belt moves a 2 mm layer of water at 10 m/sec. Assuming a Couette flow (i.e., a linear velocity profile) at 10 °C, calculate the necessary horsepower.

Solution:

1. Note that the power is a work per time or a force times a velocity.
2. Note that the force is the shear stress exerted by the belt on the fluid. Recall the shear stress equation.

3. Determine the derivative $\frac{du}{dy}$ by assuming a linear velocity profile i.e. Couette flow.

\[
\text{Power} = \frac{\text{Work}}{\text{time}} = F_{belt} \times V
\]

\[
F_{belt} = \tau A
\]

\[
\tau = \mu \frac{du}{dy}
\]
5. Newtonian and Non-Newtonian Fluids

- A fluid is said to be __________ if $\mu$ is independent of the velocity gradient, hence, $\tau$ varies linearly with $\frac{du}{dy}$.

  Examples of fluids are water, air, alcohol, gases and most petroleum products, $\mu$ is practically independent of the velocity gradient.

- A fluid is said to be ________________ if $\mu$ depends on $\frac{du}{dy}$.

  In general, solutions containing long chain polymers, as well as blood, slurries and suspensions are considered ____________. Various other descriptions are associated with these sorts of fluids.
__________ fluids have viscous effects which increase as the strain rate increases. Example include medium greases, sludges, and corn starch/water mixtures.

In the limit of the shear thickening fluids are the ________, which flow easily with low viscosity at low strain rates, but then become more solid-like as the strain rate increases (such as quicksand).

__________ fluids have viscous effects that decrease as the strain rate increases. Examples are ketchup and most salad dressings.

In the limit of shear thinning fluids are ________, which behave as a solid and then a fluid (ie. they don't begin to flow until a finite stress has been reached - ex. toothpaste and heavy greases).

![Diagram showing types of viscous and plastic fluids.](image-url)

**Fig. 1-2.** Types of viscous and plastic fluids.
6. Descriptions of Fluid Motion

There are two different approaches that describe a fluid flow:

I) ___________ - All fluid particles are followed and their location, velocity, density, pressure, etc. found as they travel in time.

ii) ___________ - The velocity, density, pressure, etc. is found at all fixed points in space as functions of time. (This approach is the followed in this course).

The flow variables will be functions of the three spatial coordinates $x, y, z$, and time $t$, for example, the velocity vector $\vec{V} = f(x, y, z)$ and the pressure $p = g(x, y, z)$. While $u$, $v$ and $w$ will correspond to the velocity components in the $x$, $y$, and $z$ directions, respectively.

There are also several different ways of visualizing a flow field.

_________ Streamlines -
Think of this as the line one would get if one particle were illuminated over a period of time and a long exposure photograph was taken.
For a particle, streamlines, streaklines and pathlines coincide.

To determine an equation for the streamline consider the following diagram for the velocity vector. The slope of the \( \vec{v} \) curve is then defined by the equation

\[
\frac{dy}{dx} = (1.1)
\]

This equation defines a line in the flow field whose tangent is always in the direction of the flow; a streamline.

Equation (1.1) can be generalized to apply in three-dimensions

\[
(1.2)
\]

Note:

a. Surfaces of solids are always streamlines. Why?
b. Fluid cannot cross streamlines. Why?
EXAMPLE PROBLEM 1.3

Given the velocity field \( \vec{V} = A \hat{i} + B \hat{j} \) determine the equation of the general streamline, streakline and path line, and sketch the flow field.

Solution:
1. Notice that the flow does not depend upon time, therefore the streamlines, streaklines and path lines coincide.

2. Use equation 1.1 to define the equation for the streamline and solve for it.

Therefore,