characteristic information allows us to say

1. $C_-$ Characteristics originating left of $x_D$ travel faster than those originating right of $x_D$.
2. $C_+$ Characteristics originating left of $x_D$ travel faster than those originating right of $x_D$.

The result of this is that left moving waves diverge and right moving waves coalesce.

The shock tube regions are typically numbered as shown below.
Expansion Waves

Expansion waves are finite waves, hence they travel at the local speed of sound. Since the $C_-$ wave just to the left of $x_D$ travels faster than the wave just to the right of $x_D$, they spread apart. Hence, the “just right” wave never catches up to the “just left” wave. Therefore, the head always travels with the left initial conditions. However, the “just right” wave travels into a region influenced by the “just left” wave. Since the expansion wave begins at a single point it is known as a centered wave.

Note that properties in region 4 are constant until the arrival of the centered expansion. Hence $J_+$ is a constant for the entire region. Then since $J_-$ is constant along the head of the expansion wave, properties along the head are constant. Therefore, properties encountered by the $C_-$ wave just behind the head are also constant. This holds throughout the entire expansion wave. For this reason all of the characteristics are straight lines and the wave is called a simple wave.

If the head encountered different properties its speed would change. The $C_-$ waves would no longer be straight lines and the regional changes would produce a nonsimple wave.

The characteristic equations can be used to determine properties within the expansion wave.

\[
J_{+4} = \text{const} \quad \quad \quad u + \frac{2a}{\gamma - 1} = u_4 + \frac{2a_4}{\gamma - 1} \tag{7.60}
\]

Rearrange $u_4 = 0$

\[
\frac{a}{a_4} = 1 - \frac{\gamma - 1}{2} \frac{u}{a_4} \tag{7.61}
\]

Perfect gas

\[
\frac{T}{T_4} = \left[1 - \frac{\gamma - 1}{2} \frac{u}{a_4}\right]^\frac{p}{\gamma - 1} \tag{7.62}
\]
Isentropic relations

\[
\frac{p}{p_4} = \left[ 1 - \frac{\gamma - 1}{2} \left( \frac{u}{a_4} \right)^2 \right]^{\frac{2}{\gamma - 1}} \quad (7.63)
\]

\[
\frac{\rho}{\rho_4} = \left[ 1 - \frac{\gamma - 1}{2} \left( \frac{u}{a_4} \right)^2 \right]^{\frac{2}{\gamma - 1}} \quad (7.64)
\]

Now since \( C_\infty \) waves are straight lines

\[ x = (u - a)t \quad (7.65) \]

So that \( u = \frac{2}{\gamma + 1} \left( a_4 + \frac{x}{t} \right) \quad - a_4 \leq \frac{x}{t} \leq u_3 - a_3 \quad (7.66) \)

Within the wave

To completely solve these equations for the shock tube problem we need \( u_3 \) and \( a_3 \). However, by knowing that \( u_3 = u_p \) (7.62) allows the determination of \( T_3 \) and hence \( a_3 \). Unfortunately, the shock properties were based on the pressure ratio across the shock and not the initial conditions. Using the fact that \( p_3 = p_2 \) across the contact discontinuity and (7.63) gives a relation for the pressure in region 2 in terms of pressures in regions 1 and 4.

**Contact Discontinuity**

The above makes use of the fact that for a contact discontinuity

\[
p_2 = p_3 \quad u_2 = u_3
\]
Reflected Expansion Wave

The reflected expansion wave is nonsimple. Note that once the head is reflected it encounters a continuous variation in properties, hence in $J_-$ values. The $J_+$ value of the reflected wave is fixed by the incoming $J_-$ and the wall boundary condition.

Although it was possible to develop simple expressions for the simple region it is very difficult to do so for the nonsimple region. We can instead resort to a crude method of characteristics by taking discrete characteristics and following their progress through reflection.

Process

1. $u$ and $a$ are known along the incoming $C_-$ waves, since $J_+$ is the same throughout the simple wave.

2. A $C_-$ wave becomes a $C_+$ wave through reflection. The incoming $C_-, J_-$ and the $u=0$ boundary condition determine $J_+$.

3. The earlier method is used to determine properties along the incoming and outgoing characteristics by noting that $J_-$ and $J_+$ are still constant along $C_-$ and $C_+$ waves.

Approximation: The location of the intersections since the characteristics are no longer straight lines.